Medgar Evers College Math Circle The Middle School Initiative @ The Immaculate Heart of Mary Middle School Brooklyn New York

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- To draw you to mathematics and to motivate you to excel in this subject
- To encourage you to undertake a future linked with mathematics, whether as mathematicians, mathematics educators, scientists, computer scientists, economists or business leaders.

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- I am going to introduce you to an area of mathematics called Number Theory

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- It is an excellent tool for exploring number theoretic questions.

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- By actively participating in the development of the topics we develop a solid understanding of the material and gain valuable early insights into the realities and opportunities of mathematical research.

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Definition

An integer p is **prime** if $p \geq 2$ and the only positive divisors of p are 1 and p. An integer n is **composite** if $n \geq 2$ and n is not prime.

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- Can you list all of the prime numbers up to 50?
- $\{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47\}$
- Can we run out of primes? I.e. do they ever stop appearing?

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- It is prime for all $n \geq 1$.

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- Are there infinitely many perfect numbers?
- Is there a fast algorithm for factoring large integers? [A truly fast algoritm for factoring would have important implications for cryptography and data security.]

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- Show that the square of an odd number is an odd number.
- Show that the product of an odd number and an even number is an even number.

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